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ENGINEERING GRAPHICS SEMINAR  
GRAPHICAL PLOTTING OF DATA.  
AN APPLICATION OF THREE AND FOUR  
DIMENSIONAL THEORETICAL DESCRIPTIVE  
GEOMETRIES

C. Ernesto S. Lindgren  
Visiting Research Engineer  
United States Steel Corporation

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Department of Graphics and Engineering Drawing  
School of Engineering and Applied Science  
Princeton University

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### ABSTRACT

The paper introduces new methods of graphical plotting by means of theoretical four-dimensional descriptive geometry, re-introduces known methods derived from theoretical three-dimensional descriptive geometry and compares the latter with the usual two-dimensional plotting characterized by a third parameter to indicate the three-dimensionality of the representation.

1.

### THREE-DIMENSIONAL PLOTTING

Assume that three parameters are interrelated, and that to the given value of one it corresponds a series of values of the other two. This may be represented in a table such as that of figure 1.

TABLE 1

[illegible]

Figure 1

The usual approach of graphical representation is to plot two-dimensional diagrams, making use, for example, of a cartesian system of reference. To indicate the three-dimensionality of the variation, each diagram is characterized by the value of the third parameter, in our example,  $x$ . (Figure 2).

2.

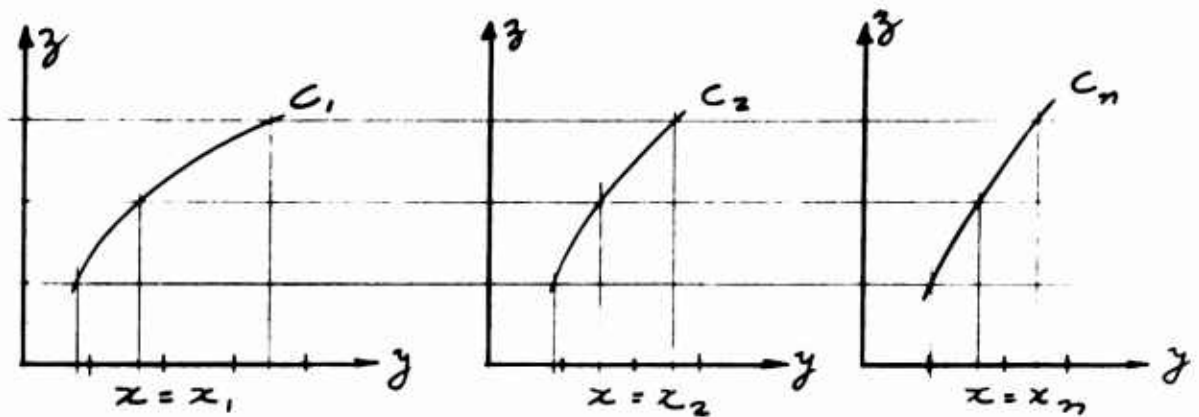


Figure 2

This representation seems to be satisfactory (if only judged by the wide use of it) but, presents one problem: interpolation. We would not care to discuss the merits of any method of interpolation, let it be intuitive, analytical or graphical, but we do care in pointing out that the average, daily user of graphical representations of this type, does not have either time or means, to make use of those intuitive, analytical or graphical methods of interpolation. Therefore, we propose to modify the way of looking at the problem, in order to arrive to a more easily understood and applicable method of interpolation.

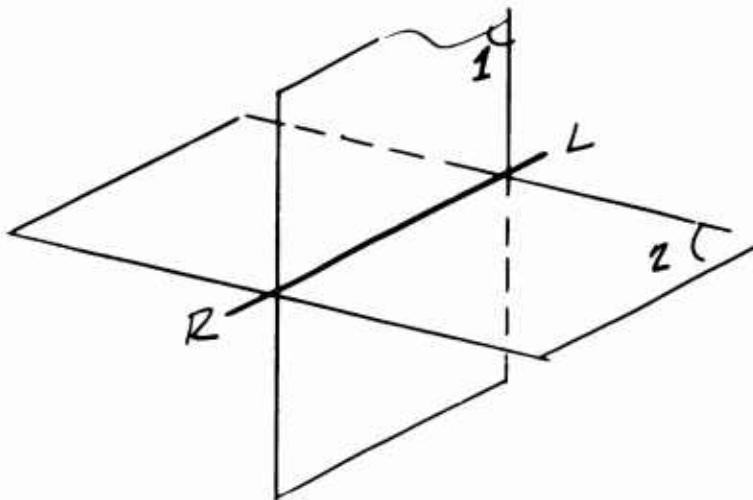
The modification is this: instead of looking at each two-dimensional diagrams as independent plane representations, let us place them together, by making use of some interesting properties of the representation of a plane through theoretical

3.

three-dimensional descriptive geometry.

This property is the following<sup>1)</sup>:

In the three-dimensional descriptive geometry, we deal with two planes, perpendicular to each other, which form the system of reference. A pictorial view is shown in figure 3.



Plane 1  $\perp$  Plane 2

$(\text{Plane 1}) \times (\text{Plane 2}) = \text{Line}$

Figure 3

Assume now a plane  $\alpha$ , perpendicular to Plane 1 (see figure 4).

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<sup>1)</sup> For more details consult a treatise on Theoretical Three-Dimensional Descriptive Geometry (Mongean representation).

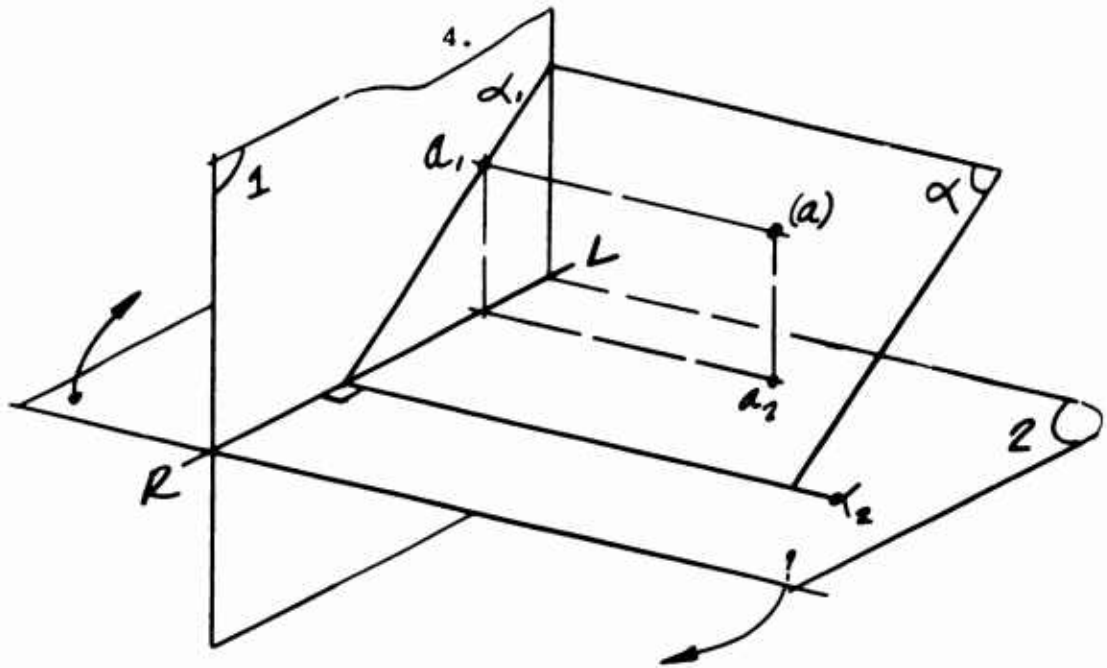


Figure 4

This plane will determine a line  $\alpha_1$ , in Plane 1 and a line  $\alpha_2$ , in Plane 2, so that  $\alpha_2$  is perpendicular to RL.

Assume next, a point (a) in this plane and through this point draw perpendiculars to Plane 1 and Plane 2. We will obtain  $a_1$  and  $a_2$ , which are the projections of (a). The interesting property that we mentioned is that  $a_1$  is found on  $\alpha_1$ . Therefore,  $\alpha_1$  is the geometric locus of the projections of all points of plane  $\alpha$  in Plane 1.

The method of three-dimensional descriptive geometry, calls for the rotation of Plane 2 about line RL, clockwise,

5.

until its superimposition on Plane 1. Therefore, we obtain the following plane representation (orthographic projections) of the plane  $\alpha$ . (Figure 5).

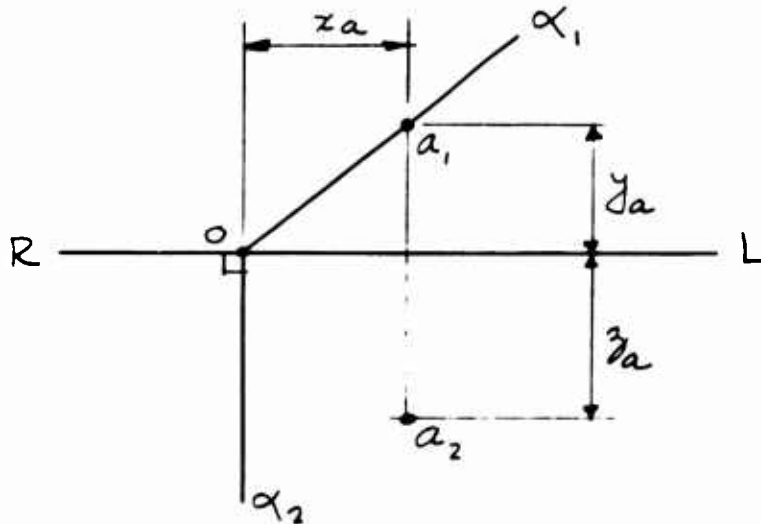


Figure 5

Now, since the point (a) is a point in the three-dimensional space, we characterize its position by three coordinates, which are identified as  $x, y, z$ , in figure 5. Notice however, that the coordinates  $y$  and  $z$ , fix the value of coordinate  $x$ . Therefore, we may disregard  $x$  and operate only with  $y$  and  $z$ , to locate any point of the plane, as long as we know the angle between  $\alpha_1$  and RL. (See figure 6).

6.

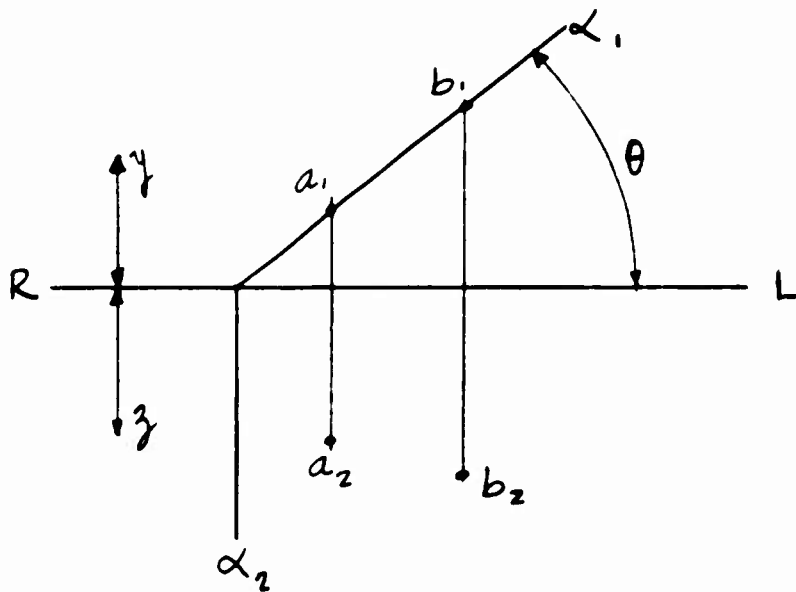


Figure 6

Let us look back at figure 2 and conclude that the angle  $\theta$  may be associated to the coordinate  $x_1$ .

Therefore, we may arrive to the following type of representation of the points which coordinates are given in Table 1 (Figure 1). See figure 7.

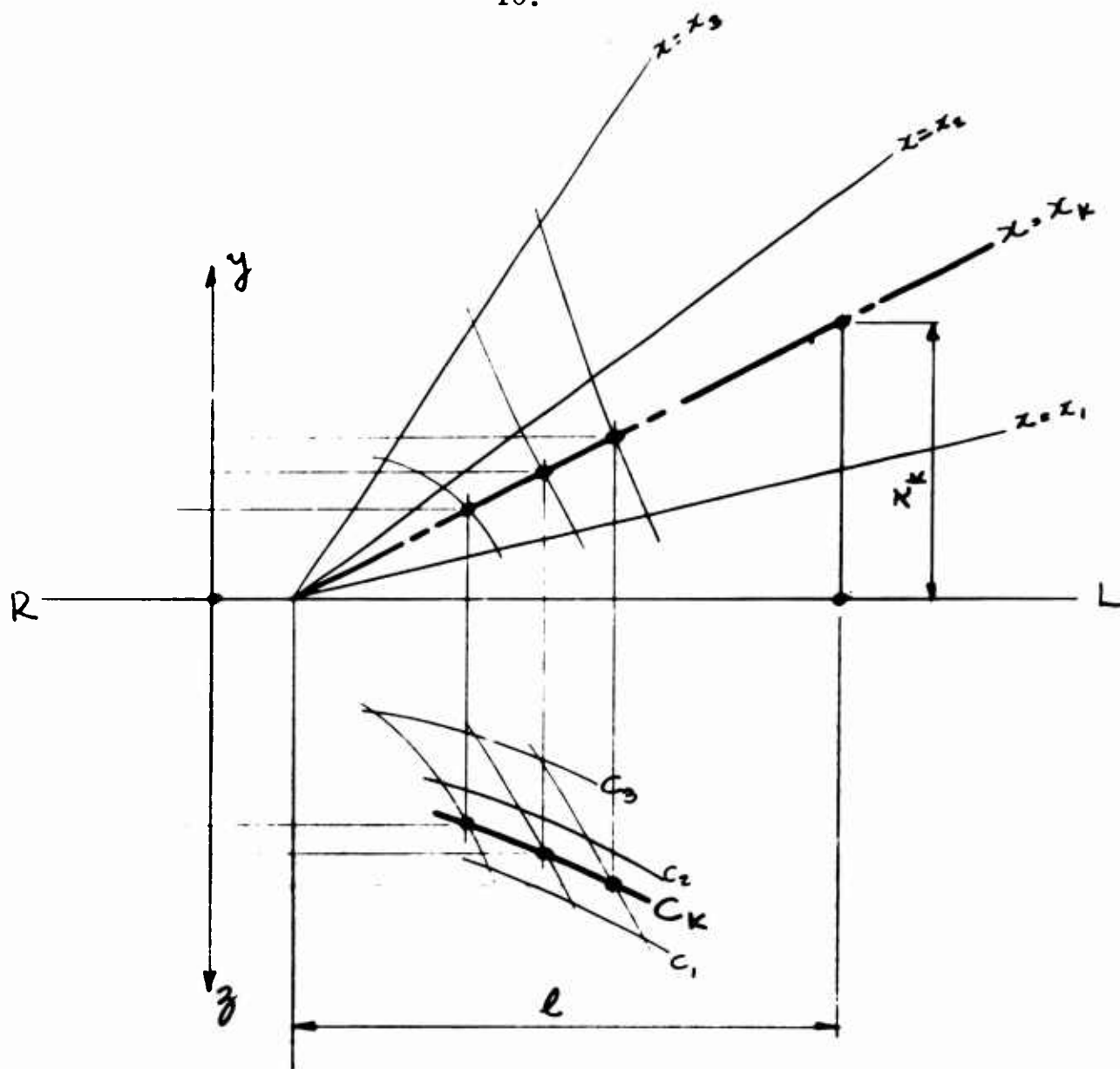


This representation, as it stands, allows the reading of the coordinates of any point in the curves  $C_1, C_2, C_3, \dots, C_n$ , as rapidly as in the mode of representation of figure 2.

However, this is not all. Notice that the curves  $C_1$  are generatrices of a surface, in the three-dimensional space, and that the representation as shown in figure 2, indicates the result of a plane section in that surface. The variation between two consecutive sections is not so immediately obvious. But, in figure 7, we may obtain, immediately, the configuration of that variation, as shown in figure 8, by simply drawing the curves connecting the corresponding points of each plane. Interpolation is, therefore, easily made, as shown in figure 9.



10.



Interpolation

for  $x = x_k$

Figure 9

11.

In figure 10 we indicate the application of another method of descriptive geometry, namely, rotation, and the result obtained can also be used for interpolations.

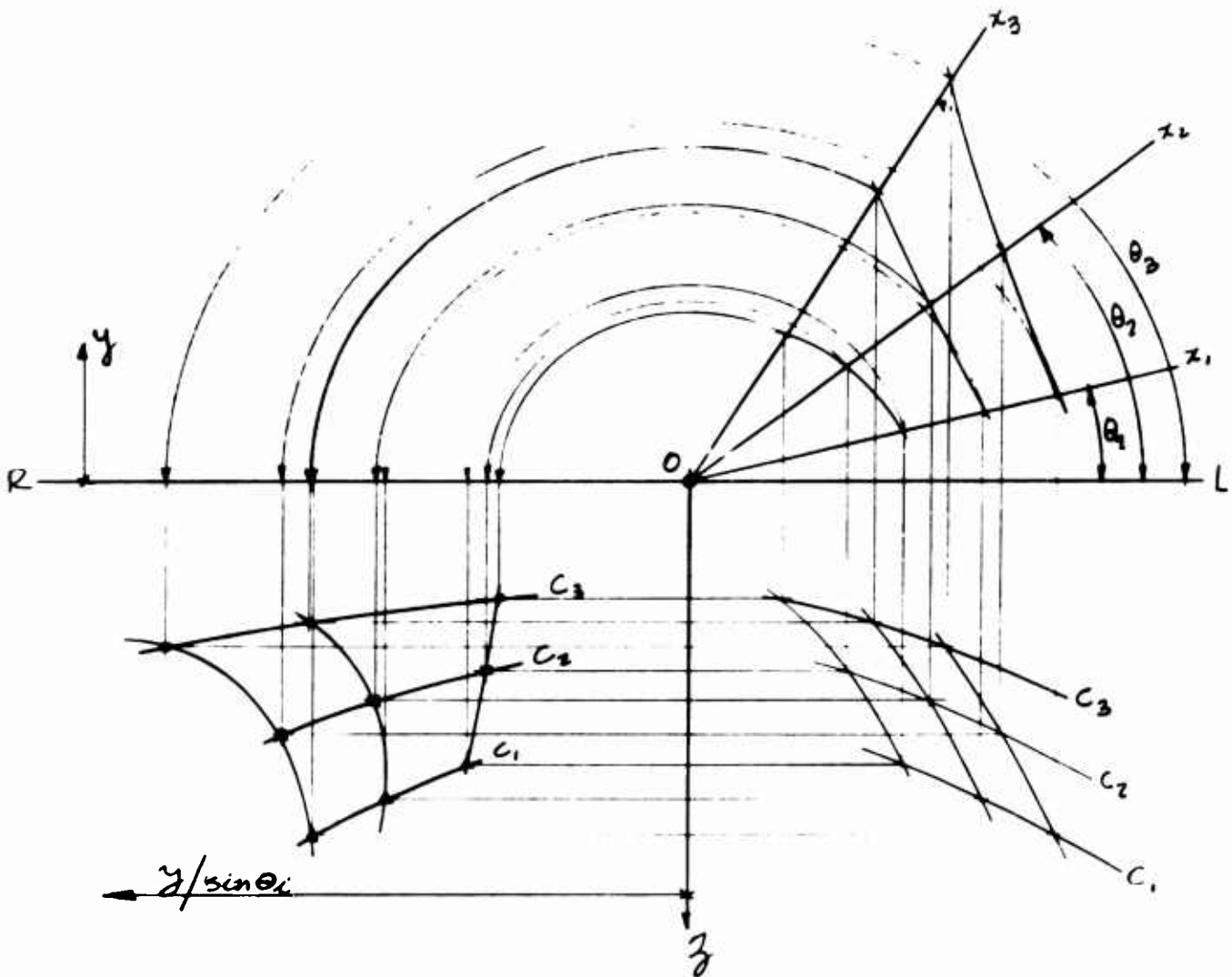


Figure 10

12.

A variation of the representation used in figure 8 is indicated in figure 11. The method of rotation has been applied.

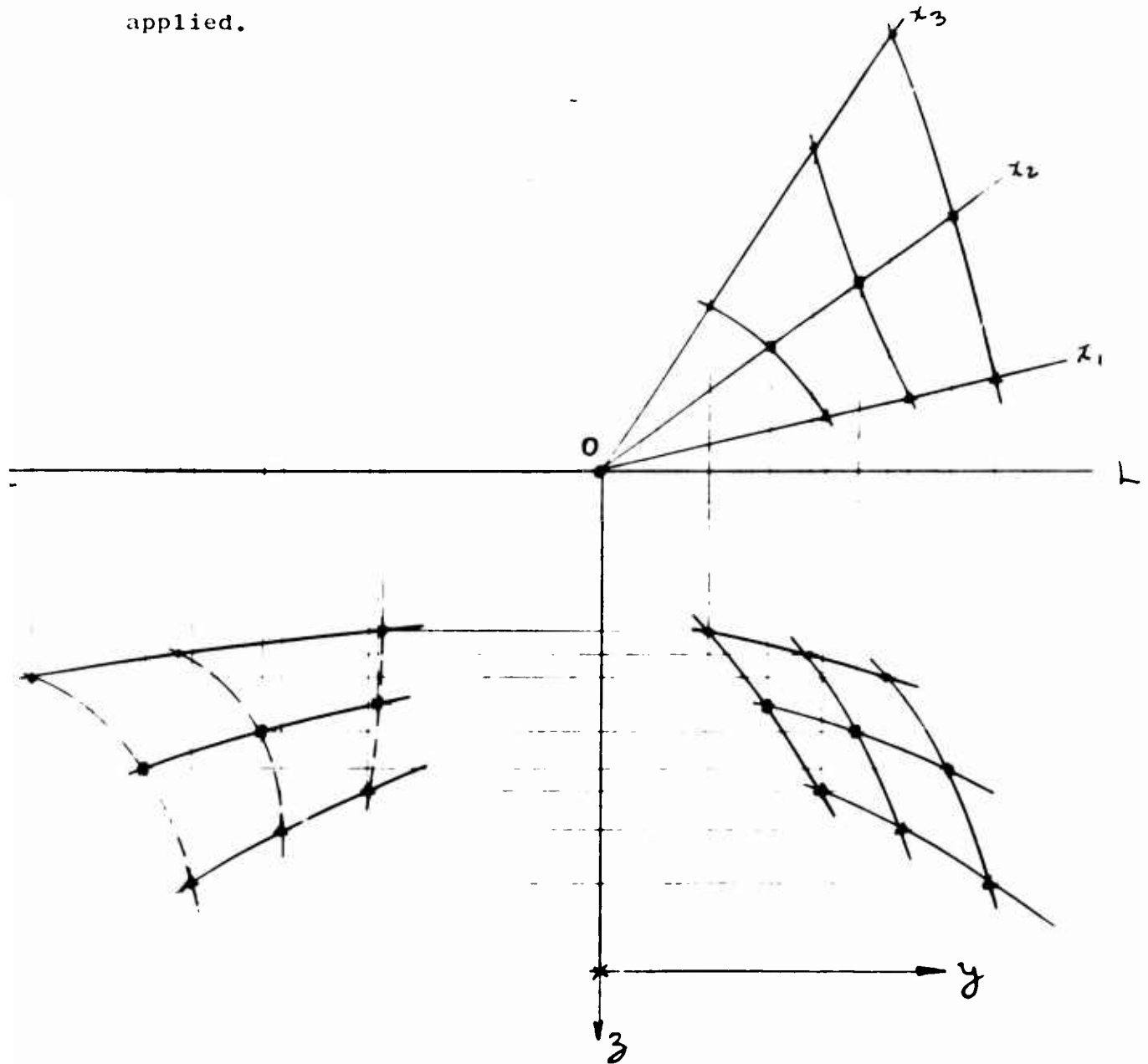


Figure 11

The representation just presented, by means of three-dimensional descriptive geometry, is the simplest one, involving three variables or parameters. Because the representation of a plane, in descriptive geometry, can have any degree of sophistication that we may think of, so will the plotting of a curve or a group of curves in one of these planes. Naturally, we should make use of these particular position, with a purpose. For instance, in cases of extrapolations, it will be more desirable to use a plane occupying a very general position in relation to the planes of the system of reference. This will permit the use of certain properties in the plane that will help to reduce somewhat the guessing of how the variation is extended beyond the boundaries of the plotted curves.

Let  $\beta$  be a plane referred to the system of the two perpendicular planes. (figure 12).

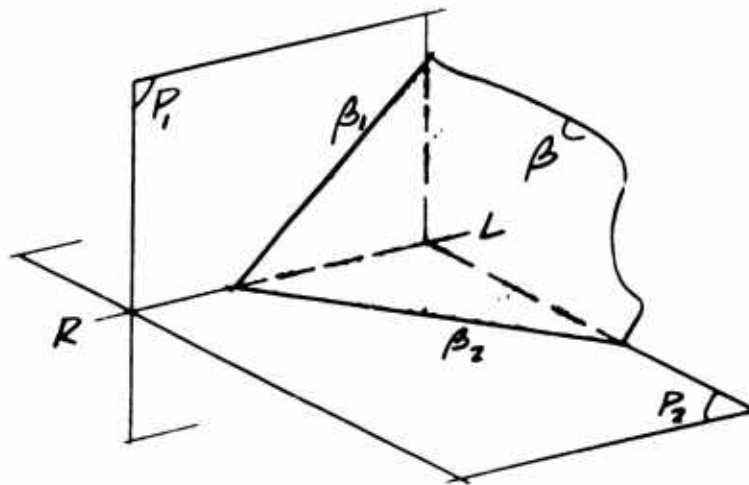


Figure 12

### 13A

The orthographic projections are shown in figure 13 and we have also indicated two points (a) and (b), of the plane, belonging to lines parallel to  $P_1$  and  $P_2$ .

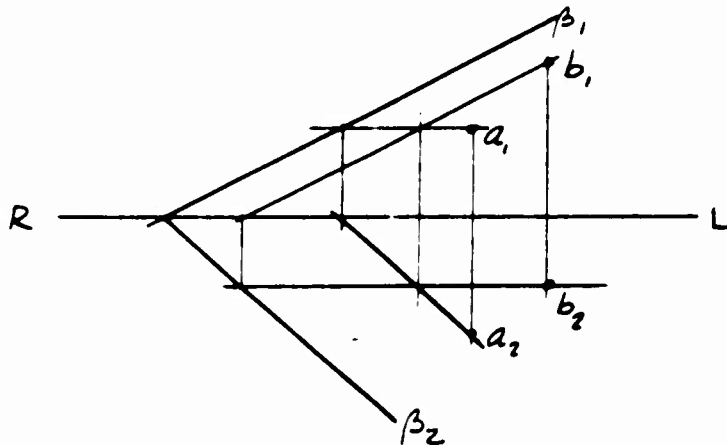


Figure 13

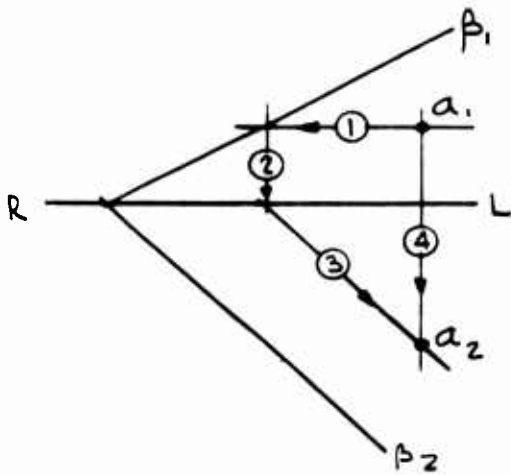
Since we have no intention of going into the details of demonstrating the conditions of belonging between the points (a) and (b) and the plane  $\beta^2$ ), nevertheless we indicate, in figures 14-a, b, and 15-a, b, how to correlate the projections of a point of a plane.

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<sup>2)</sup> Consult a treatise on Theoretical Descriptive Geometry (Mongean representation).

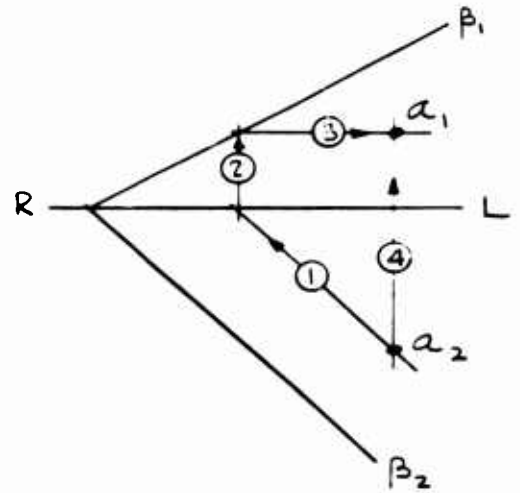


14.



GIVEN:  $a_1$

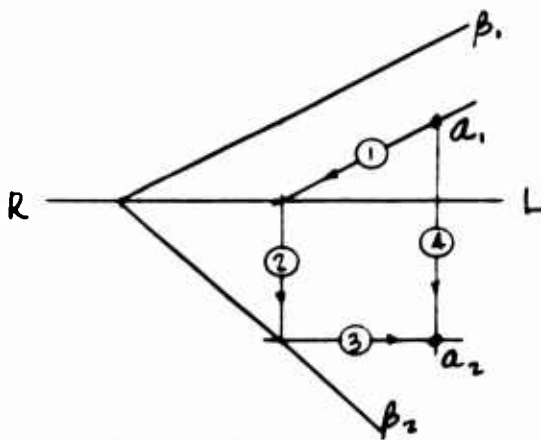
(a)



GIVEN:  $a_2$

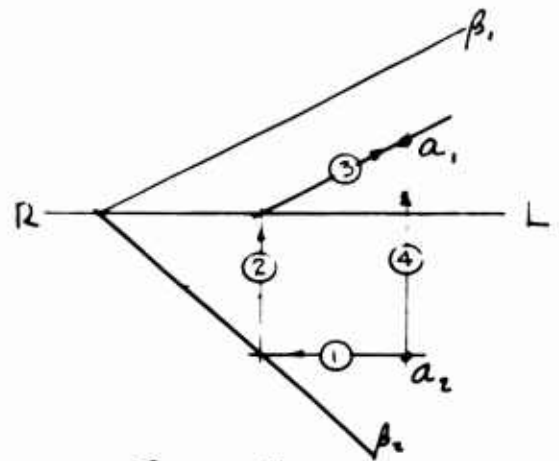
(b)

Figure 14



GIVEN:  $a_1$

(a)



GIVEN:  $a_2$

(b)

Figure 15

Properties

- a) Take two points in the plane, (a) and (b) and determine the points (x) and (y) as indicated in figure 16.

Property: the points (x), (y), and (k) are colinear, and the two projections,  $a_1b_1$  and  $a_2b_2$ , of any line of this plane will be concurrent on a point of that line. (Figure 17).

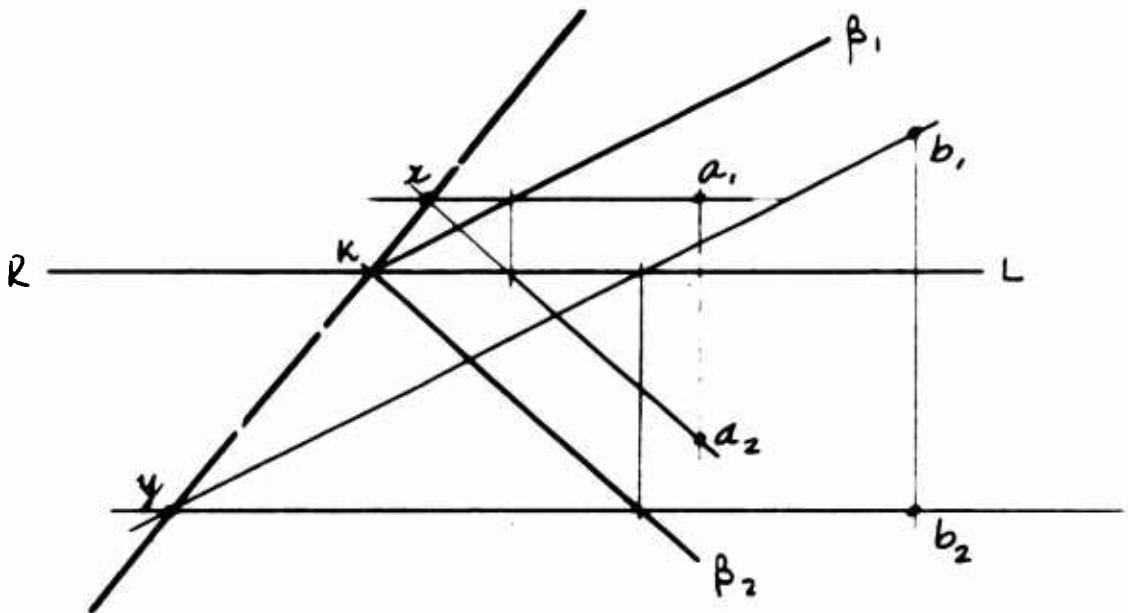


Figure 16

16.

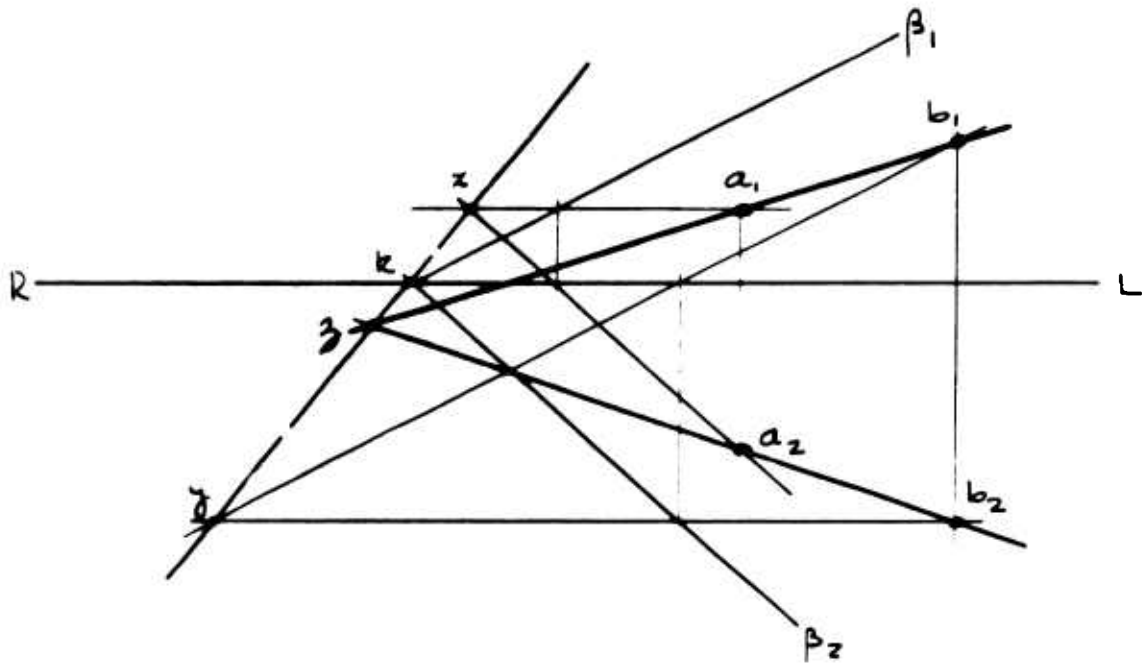


Figure 17

b) In figure 18

- (1) through  $m_2$  draw perpendicular to  $\beta_2$ ;
- (2) radius  $(km_1)$ , center in  $(k)$ , cut the perpendicular in  $(m)_1$ ;
- (3) draw  $(k) - (m)_1$ ;
- (4) draw perpendicular through  $n_2$  to  $\beta_2$  and determine  $(n)_1$  in  $(\beta_1)_1$ ;
- (5) draw parallels to  $\beta_2$  through  $(m)_1$  and  $(n)_1$ ;

17.

- (6) draw perpendiculars through  $a_2$  and  $b_2$  to  $\beta_2$  and determine  $(a)_1$  and  $(b)_1$ .

Property:  $a_2 b_2$  and  $(ab)_1$  will be concurrent in a point  $(w)$  of  $\beta_2$ .

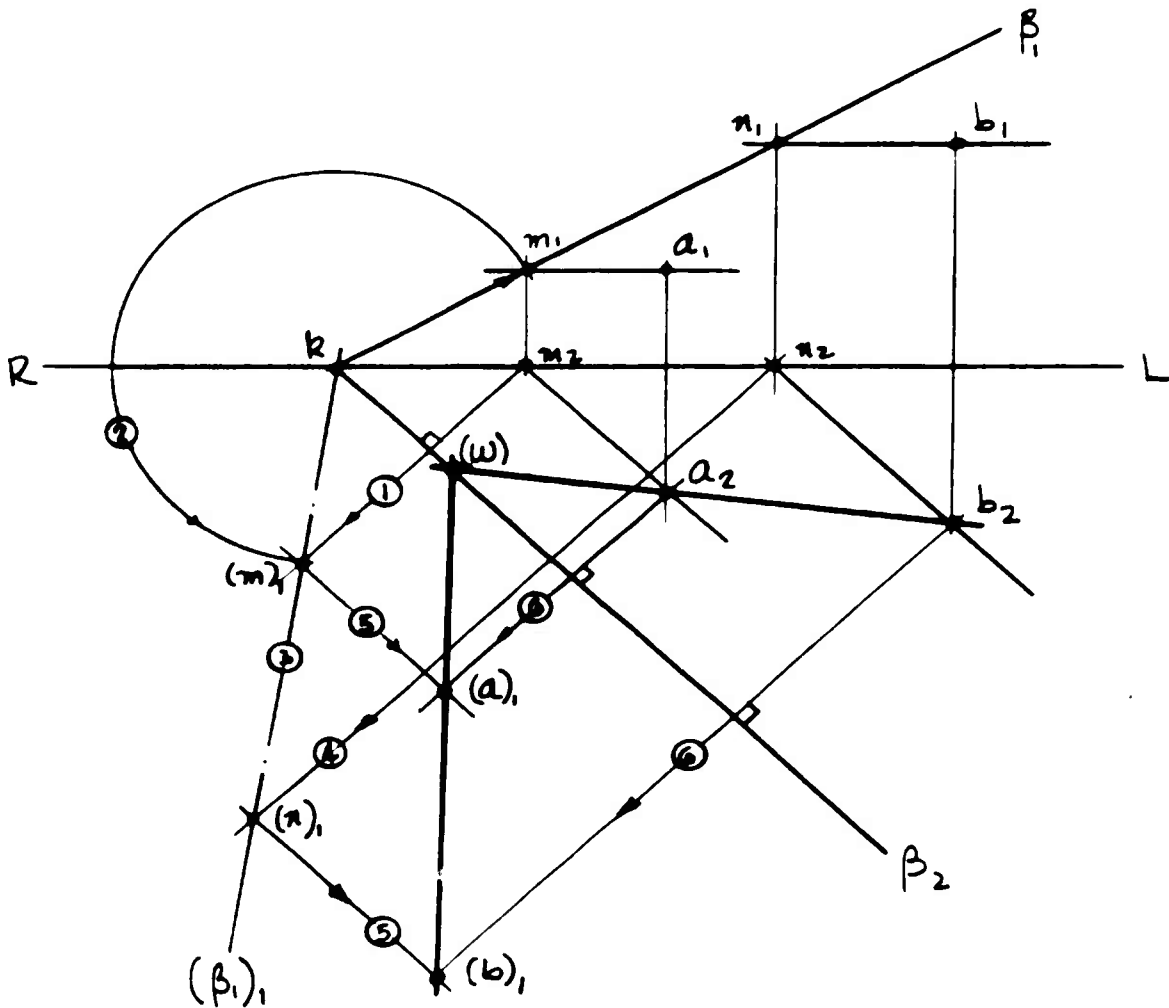


Figure 18

The use of these two important properties of the plane in the plotting of curves is as follows:

The properties also apply to any curve of the plane.

Thus, in figure 19 we show three points (a), (b), (c), of a curve in the plane and the relationships between the projections, using those properties.

Notice that any point is characterized by three coordinates. It should be kept in mind that two of the coordinates fix the value of the third (due to the two-dimensionality of the plane). In this case, we still can use this method of plotting by considering that the graduation of one of the axis of coordinates (in the example, the RL), is made in function of the projections of the point obtained by use of the two other coordinates.

19.

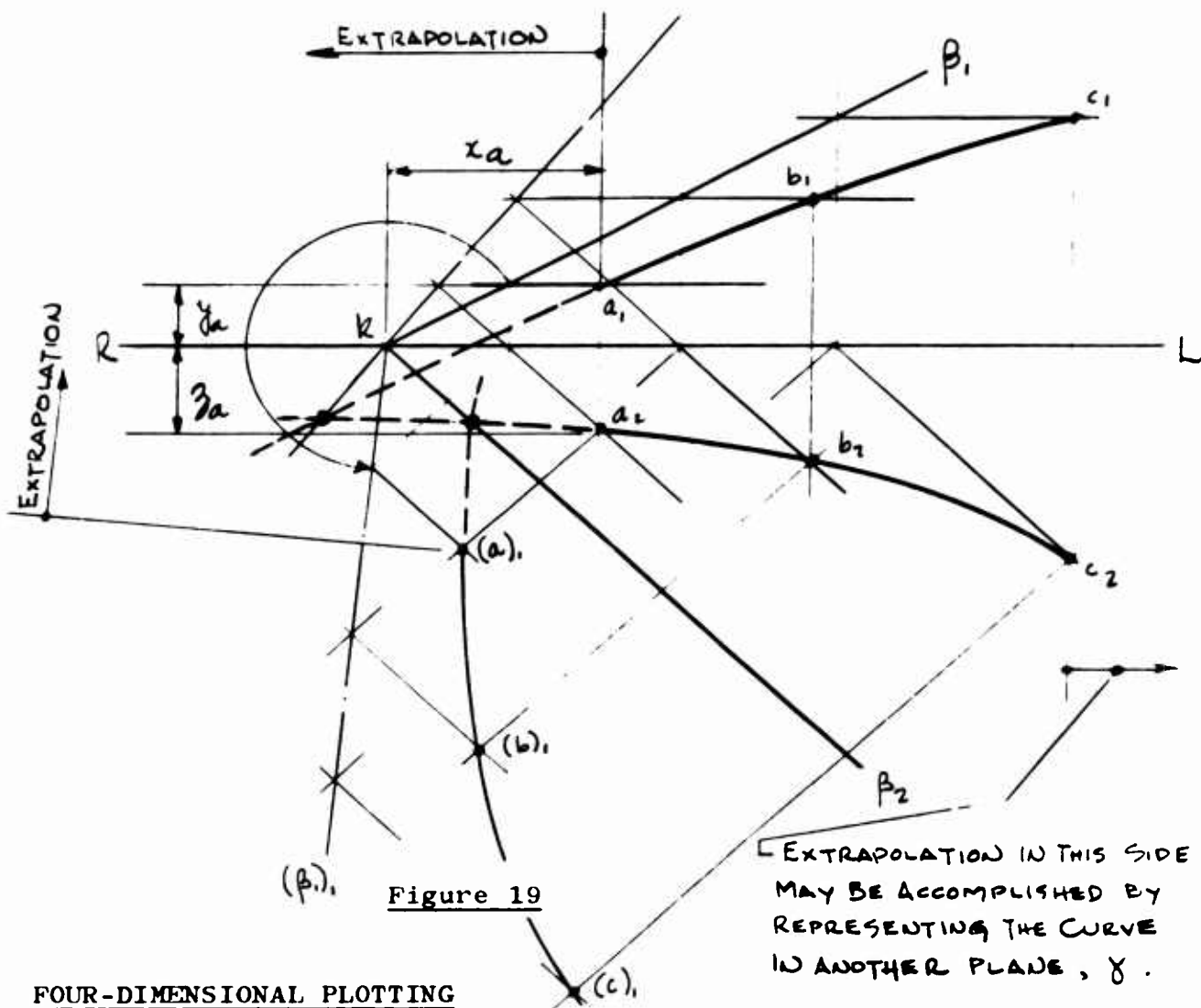


Figure 19

#### FOUR-DIMENSIONAL PLOTTING

It would be futile to try to enumerate the possible ways of plotting data by use of four-dimensional descriptive geometry. One should realize that, because of what is object of study in that geometry - namely, the 3-D space - these possibilities are a function of the number of ways that we can relate this 3-D space to the 4-D system of

reference and of the number of positions that a plane occupies within this 3-D space.

We have dealt with two of the possibilities of plane representation which should provide working tools for most of the types of data plotting. Evidently, a more developed and sophisticated analysis of theoretical descriptive geometry will provide a much more broader scope of possibilities and variation in the method of plotting.

If we now take all these possibilities and apply them to the equally broad scope of possibilities in representing a 3-D space, we would have created a somewhat large amount of means and methods of data plotting.

However, one thing must be clarified: it is not the number or even the quality of each method that is important. Important is, to have a good grasp of the mechanics of one method, based on theoretical information. This alone should permit the proper analysis and understanding of any other method, for they all are based on one common foundation: the theory of the three and four-dimensional descriptive geometries.

In discussing a method for four-dimensional plotting we will, therefore, make use of a representation of the 3-D

space which, by extension, corresponds to the representation of the plane used in three-dimensional plotting. We would recommend the consulting to some of the work done on theoretical four-dimensional descriptive geometry<sup>3)</sup>, for more information on some of the aspects of the problem, that will not be fully discussed here, and for the derivation and creation of other methods of plotting which could be found to be more appropriated to one specific problem or to one specific group of data.

The resulting representation, by orthographic projections, of a 3-D space  $\Omega$  referred to the 4-D system of reference is shown in figure 20.

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<sup>3)</sup> Contact the Department of Graphics and Engineering Drawing, Princeton University, Princeton, New Jersey.



22.

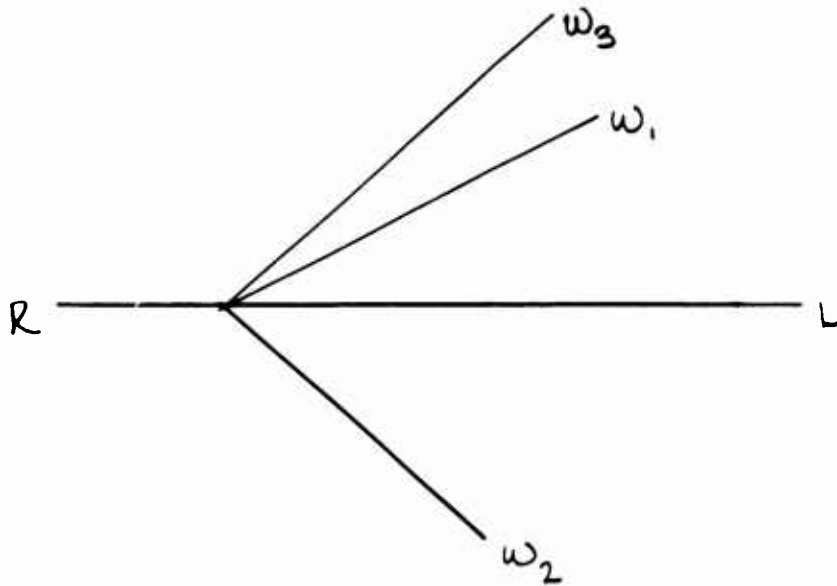


Figure 20

Here we have three planes,  $\omega_1 - \omega_2$ ,  $\omega_1 - \omega_3$ ,  $\omega_2 - \omega_3$ , represented in the same way as in three-dimensional descriptive geometry. These three planes determine the 3-D space  $\Omega$ .

By conditions of belonging between plane and 3-D space, a plane  $\alpha$ , in which one of the coordinates is constant, is represented as shown in figure 21.

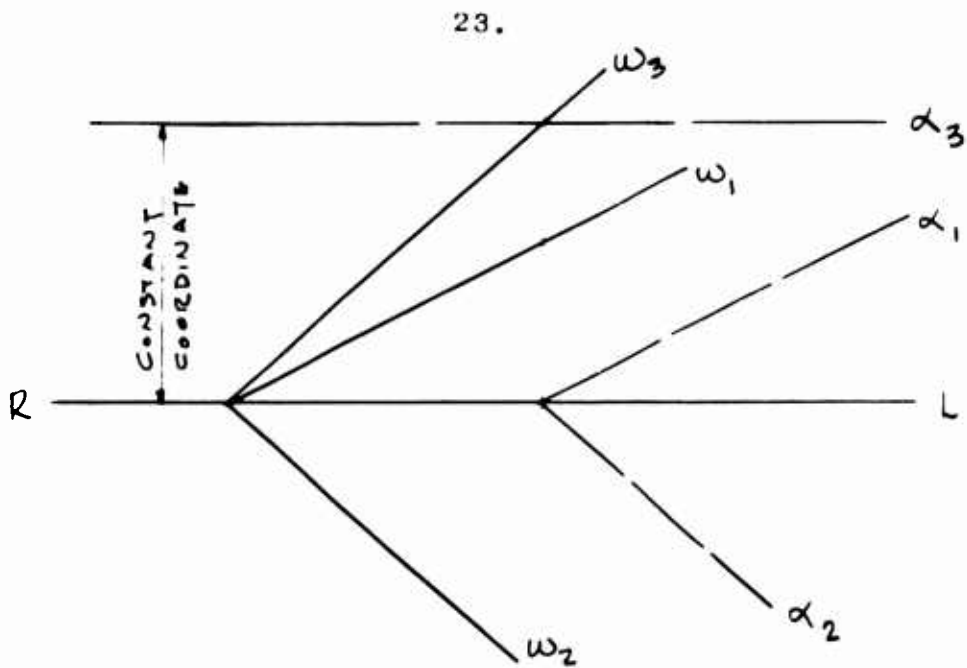


Figure 21

In this plane we can mark points, with at least two given coordinates and relate it to an arbitrary point on the reference line. Thus a point in the plane will be characterized by four coordinates. Figure 22.

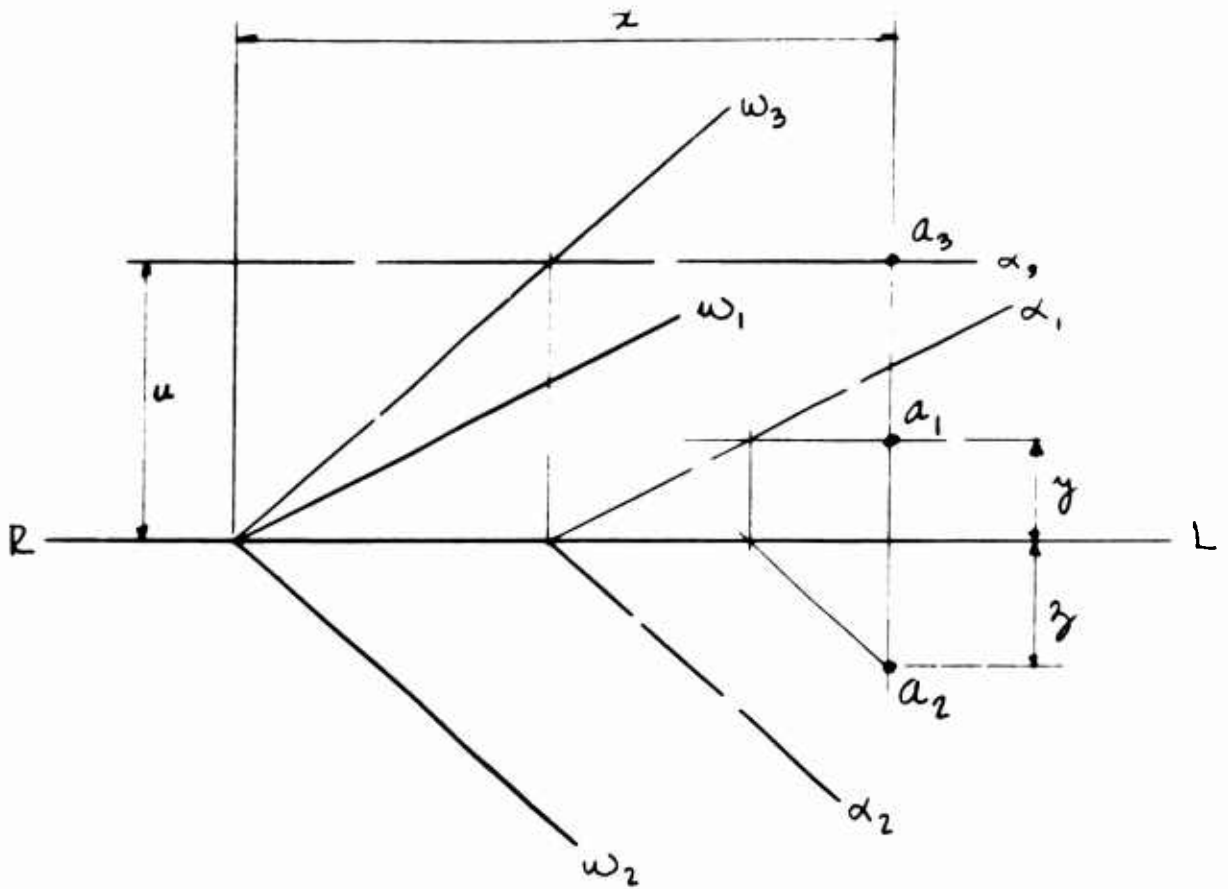
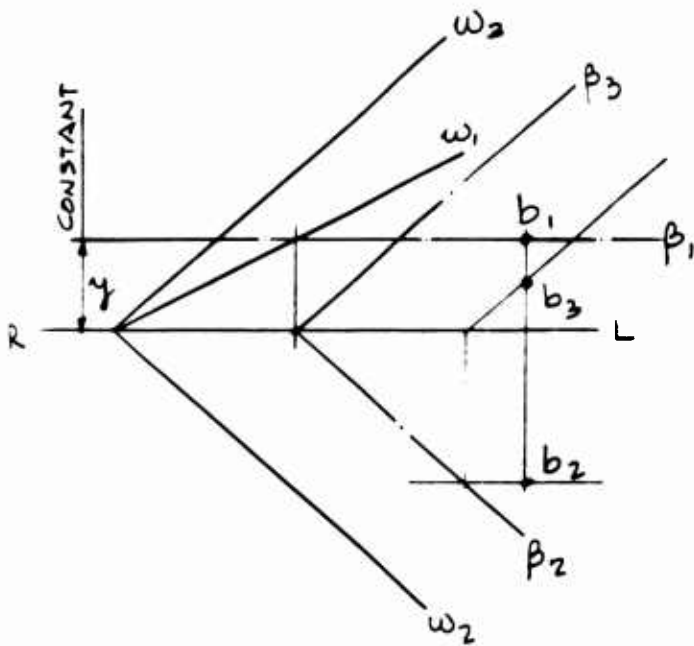
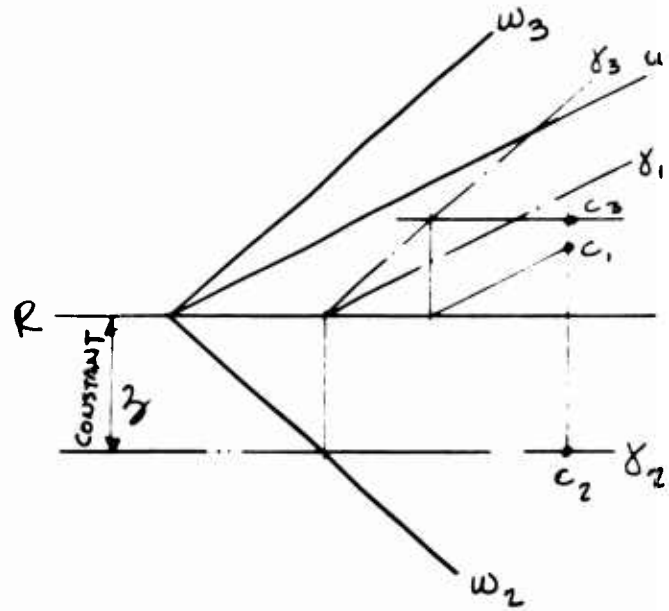


Figure 22

Furthermore, all the constructions and properties indicated in the study of the plane in the three-dimensional representation, apply to the plane  $\alpha$  and to any other plane of the 3-D space  $\Omega$ . In figures 23a, b, we show two other planes,  $\beta$  and  $\gamma$ , of the 3-D space  $\Omega$ , the first with constant coordinate  $y$  and the second with constant coordinate  $z$ .



(a)



(b)

Figure 23

With this brief presentation we can readily see that the three planes, and the curves plotted in each one, shown in figures 8, 9, and 10, can be represented in three planes,  $\mu$ ,  $\eta$ ,  $\rho$ , of a 3-D space  $\Lambda$ , as shown in figure 24.

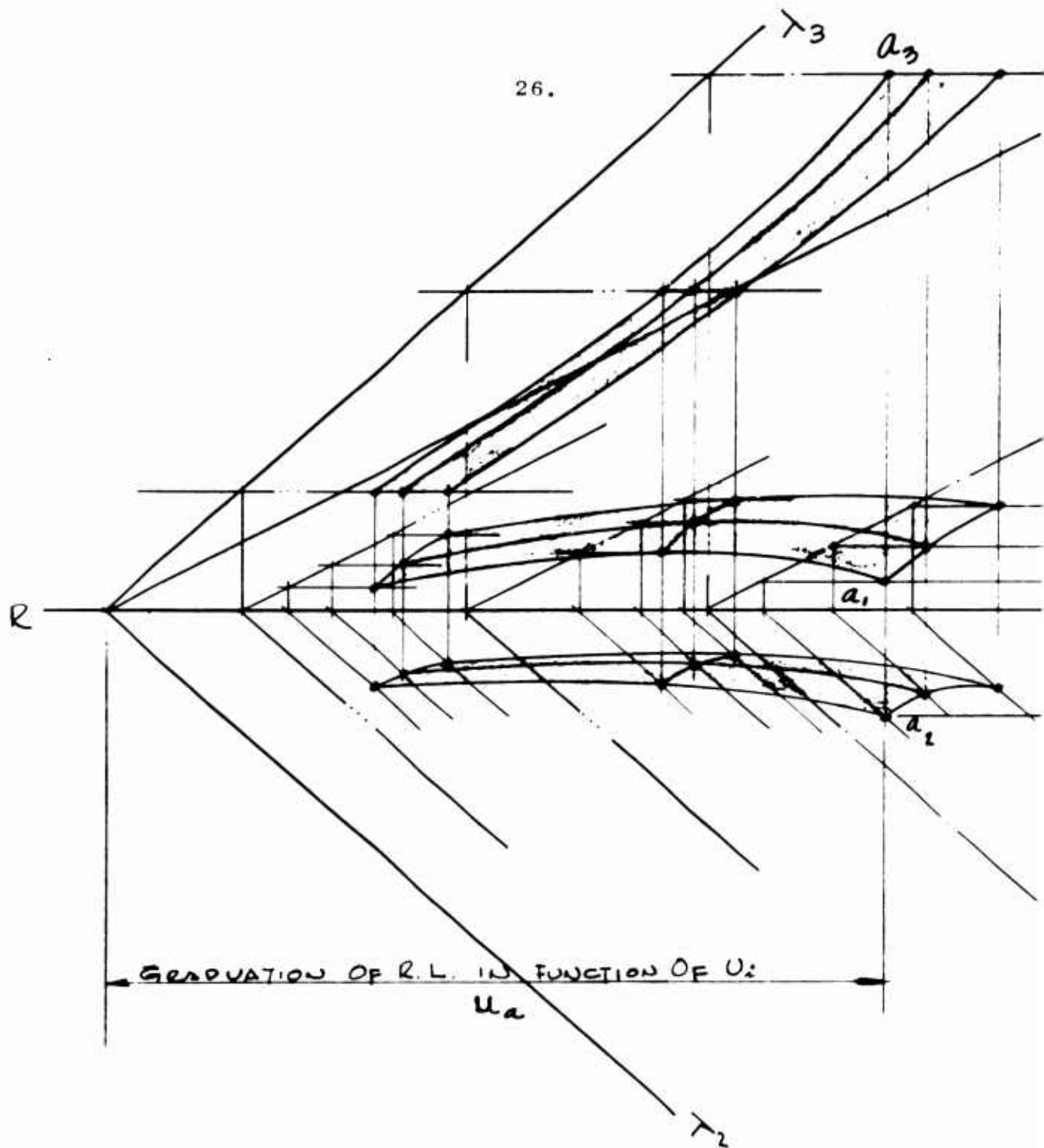
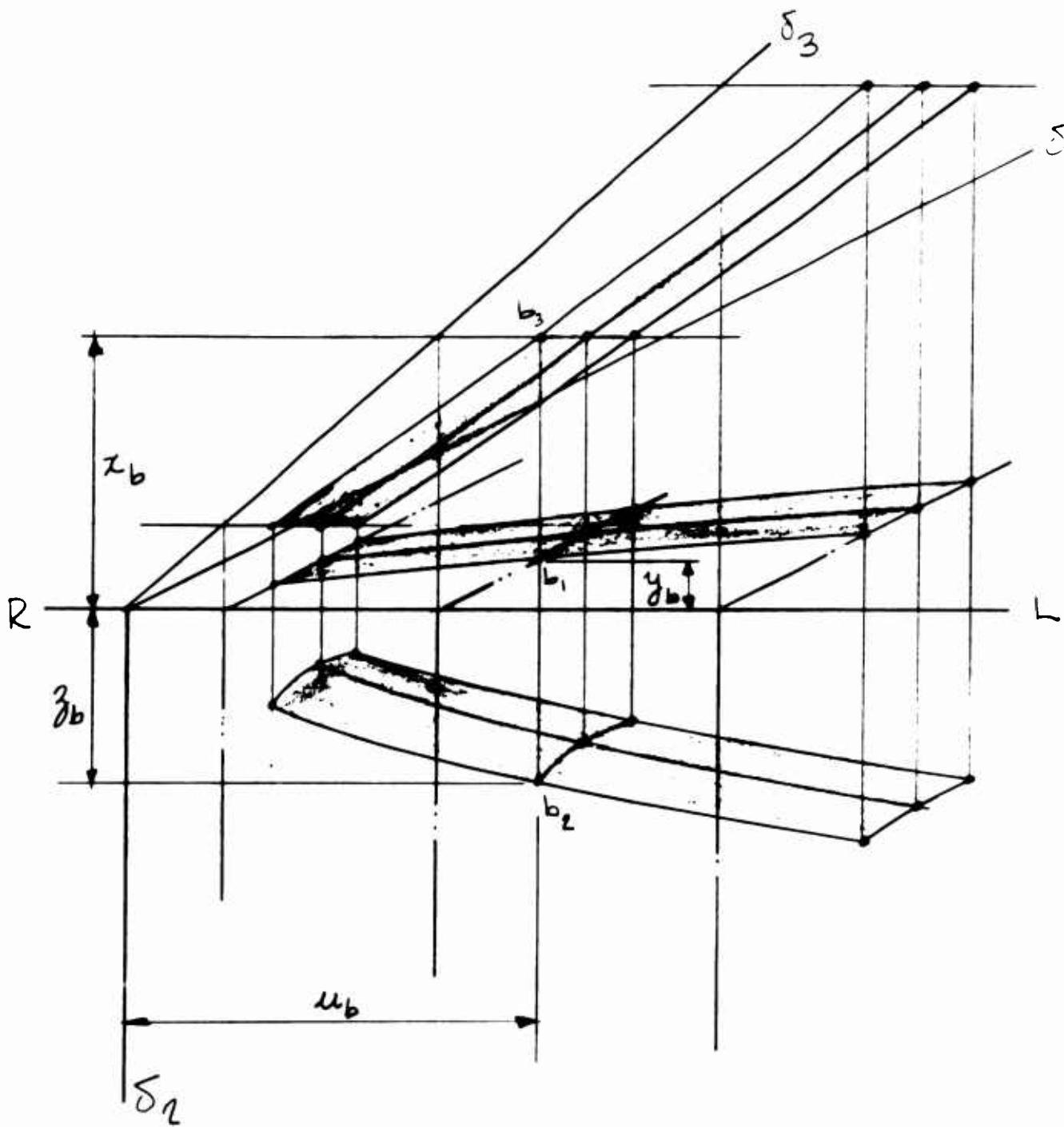


Figure 24

Since each point is characterized by four coordinates, we see that we have the possibility of assigning one more coordinate to the group of points plotted in figure 24, making the diagram, four-dimensional. Naturally, this fourth coordinate is a function of the other three, and the use of it as an independent coordinate may be made by considering that its determination in function of the other three corresponds to a graduation of the axis where it is read, in the case, the reference line.

The 3-D space  $\Delta$ , of figure 25, perpendicular to one of the 3-D spaces in the 4-D system of reference may also be used for the plotting of the points shown in figure 24. Its advantage is obvious.



MULTI-DIMENSIONAL PLOTTING

To plot multi-dimensional diagrams we have two choices:

- 1) Use a necessary number of "spaces" with one dimension less than the number of variables or parameters and "connect" the corresponding points. This corresponds to "translating" a given "space" to generate another, with one more dimension.
- 2) Use the descriptive geometry method for the defined multi-dimensional space.

Example:

Take two or more planes, represented by three-dimensional descriptive geometry and in each, plot the points of a curve, characterized by a constant coordinate. By connecting the corresponding points, we will obtain a three-dimensional representation of the variation. (Figure 26).

Compare this representation with the one shown in figure 25.



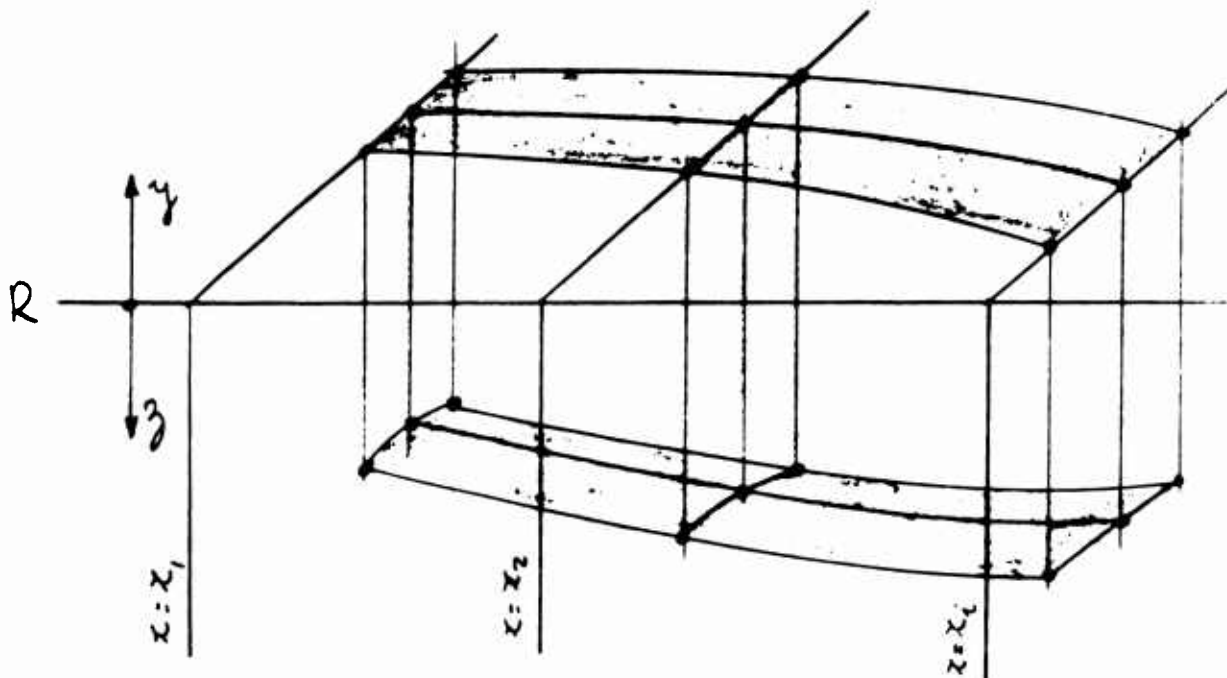


Figure 26

In figure 26, the first choice is applied. All representation is based on the study of "two-dimensional spaces" (planes) while in the figure 25, we used the second choice, with the application of a four-dimensional descriptive geometry where it is possible to analyze the "three-dimensional space" defined by the variation.

The important thing to notice is that the representation as shown in figure 26, does not allow the introduction of a fourth coordinate. In figure 25, this is perfectly possible.

31.

So, figure 26 is, in essence, a three-dimensional diagram, while figure 25 is a four-dimensional one, incorporating the three-dimensional diagram of figure 26.

Suppose then, that we want to express graphically, the relationships among a group of points characterized by say, five coordinates. What would be the proper choice? Well, the answer is a function of the nature of the coordinates. If there are no groups of points characterized by four variables and one constant parameter, we have but one choice: use the method of a six-dimensional descriptive geometry. This method in itself is not very difficult to develop as we will show, by representing a five-dimensional space referred to a 6-D system of reference.

Let us assume, however, that the points are characterized as indicated in Table 2, figure 27.

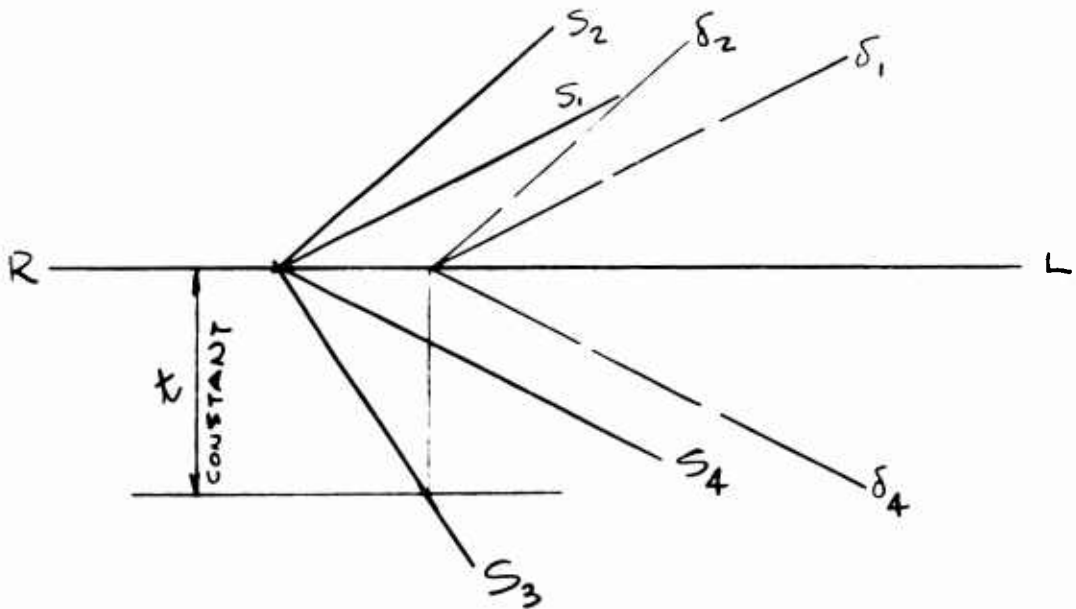
TABLE 2

	$x = x_1$				$x = x_2$					$x = x_n$			
	$y$	$z$	$u$	$t$	$y$	$z$	$u$	$t$		$y$	$z$	$u$	$t$
POINT A:													
POINT B:													
POINT C:													

Figure 27

The solution is obtained by representing each group of points,  $a_i, b_i, c_i, \dots, n_i$ , in as many four-dimensional spaces as required (function of the number of values for the parameter  $x_i$ ), and "connecting" the corresponding points. The resulting diagram will encompass five coordinates.

Undoubtedly we will need the development of a five-dimensional descriptive geometry, and the task is a major one. However, in the outcome, the graphical representation by orthographic projections of a four-dimensional space is as simple as indicated in figure 28. Here, we can characterize 3-D spaces and planes within these, and so on.



4-D space (S) - shown is a 3-D space  $\Delta$ , geometry locus of points with constant coordinate 1.

Figure 28

Thus in figures 29 and 30 we indicate how to obtain the location (projections) of a point (a) with coordinates  $x_a$ ,  $y_a$ ,  $z_a$ ,  $u_a$ ,  $t_a$ . (NOTE: the correspondence between coordinates and sub-index of the projection is arbitrary).

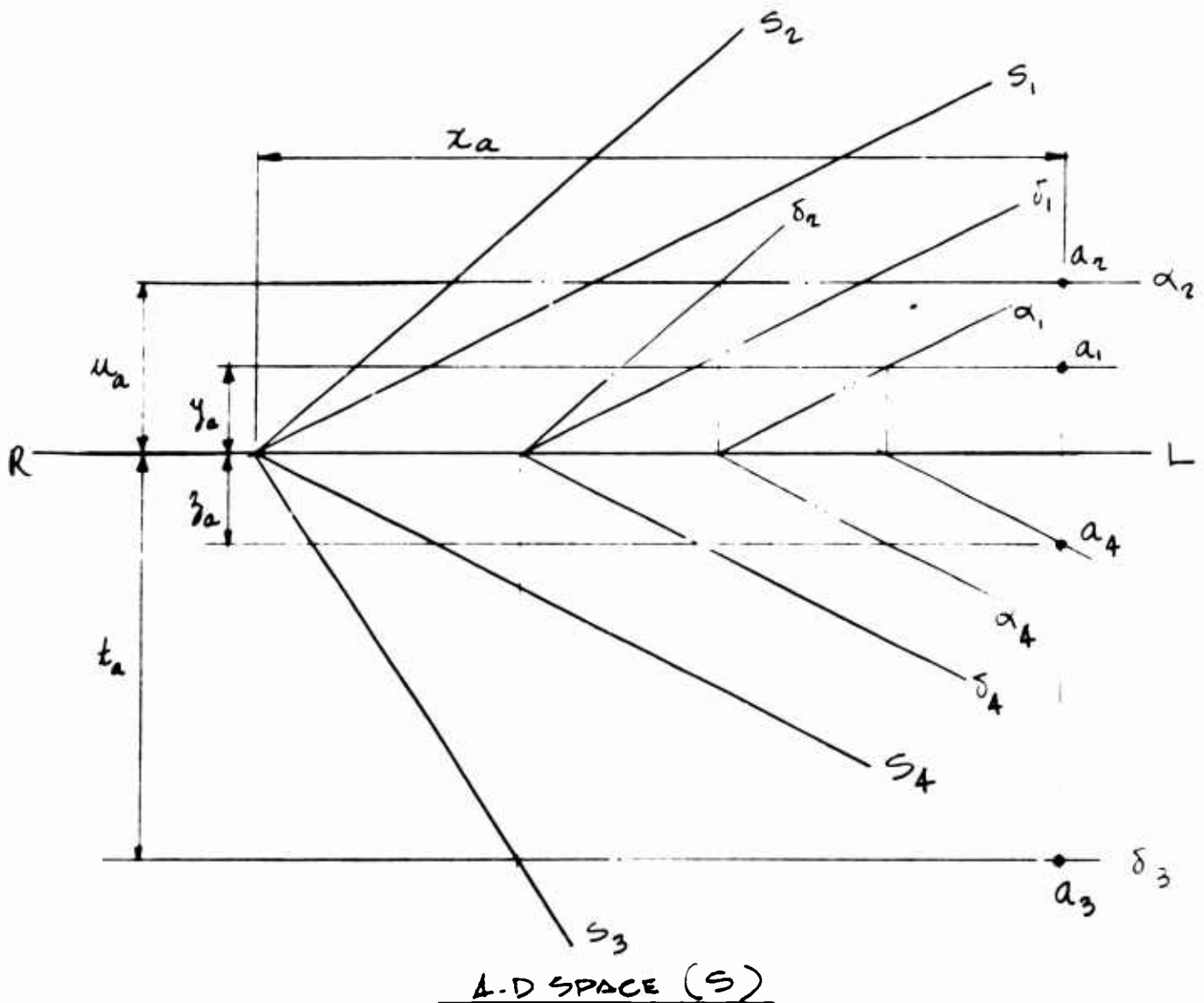
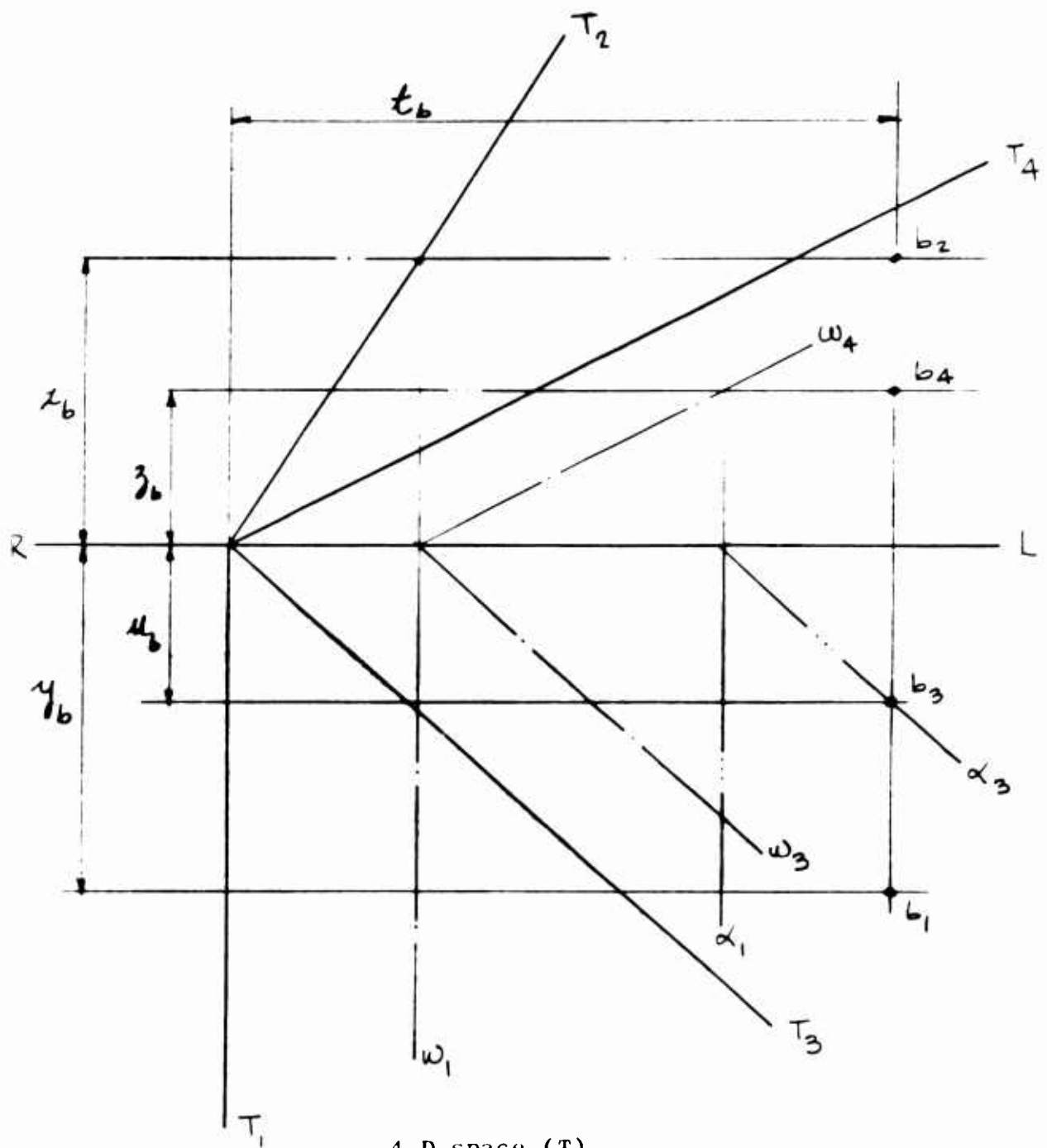


Figure 29

34.



4-D space (T)

Figure 30

Thus, to represent graphically the points of Table 2, we would consider a series of parallel four-dimensional spaces, characterized by  $x_i$ , and within each one mark the corresponding points  $a_i, b_i, \dots, n_i$ . This is shown in figure 31, where the points  $a', a'', a'''$ , are plotted in three 4-D spaces. The diagram is five-dimensional.

All points with coordinate  $x = x_1$  should be plotted in the first 4-D space; those in which  $x = x_2$ , in the second 4-D space; and so on.

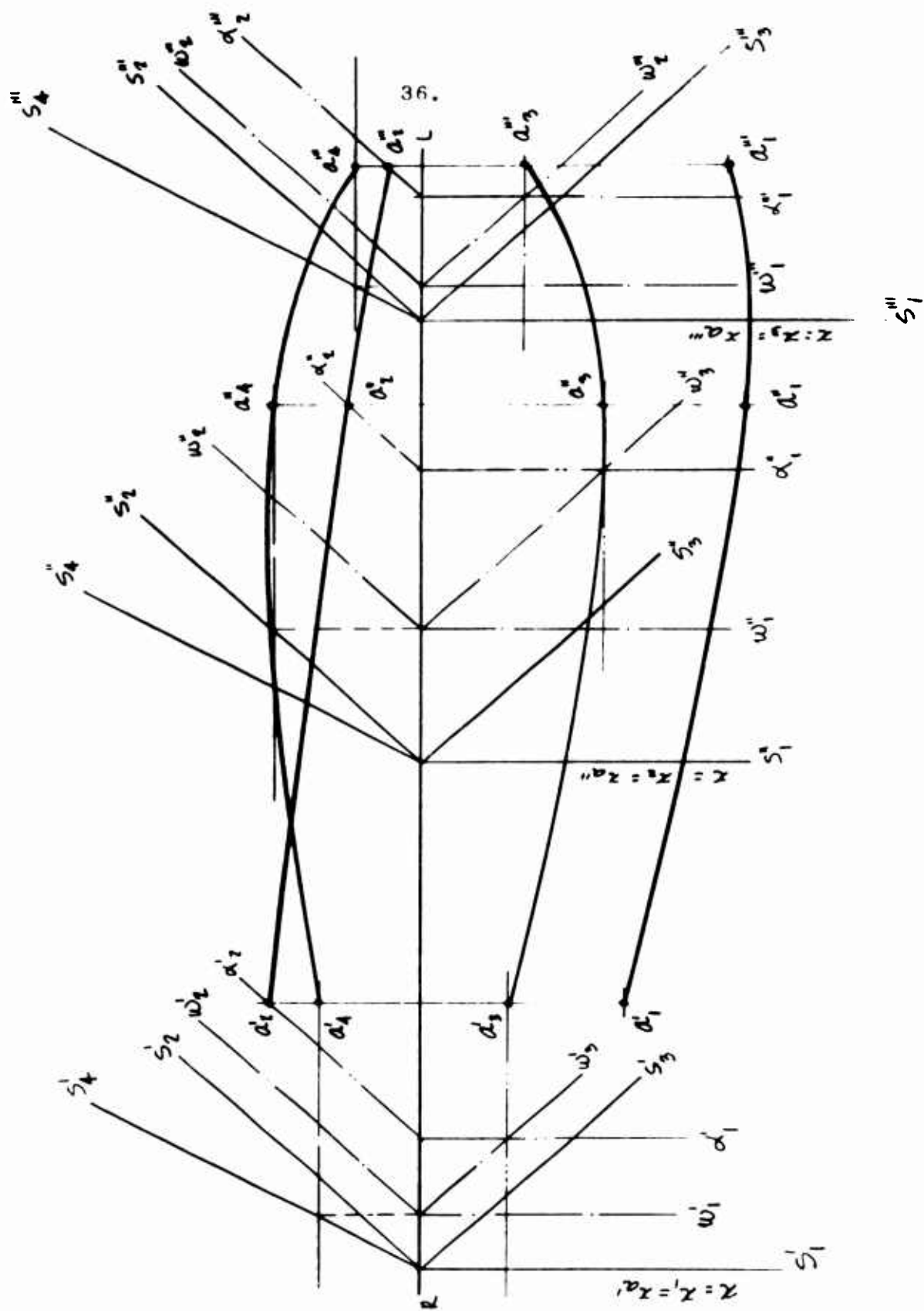
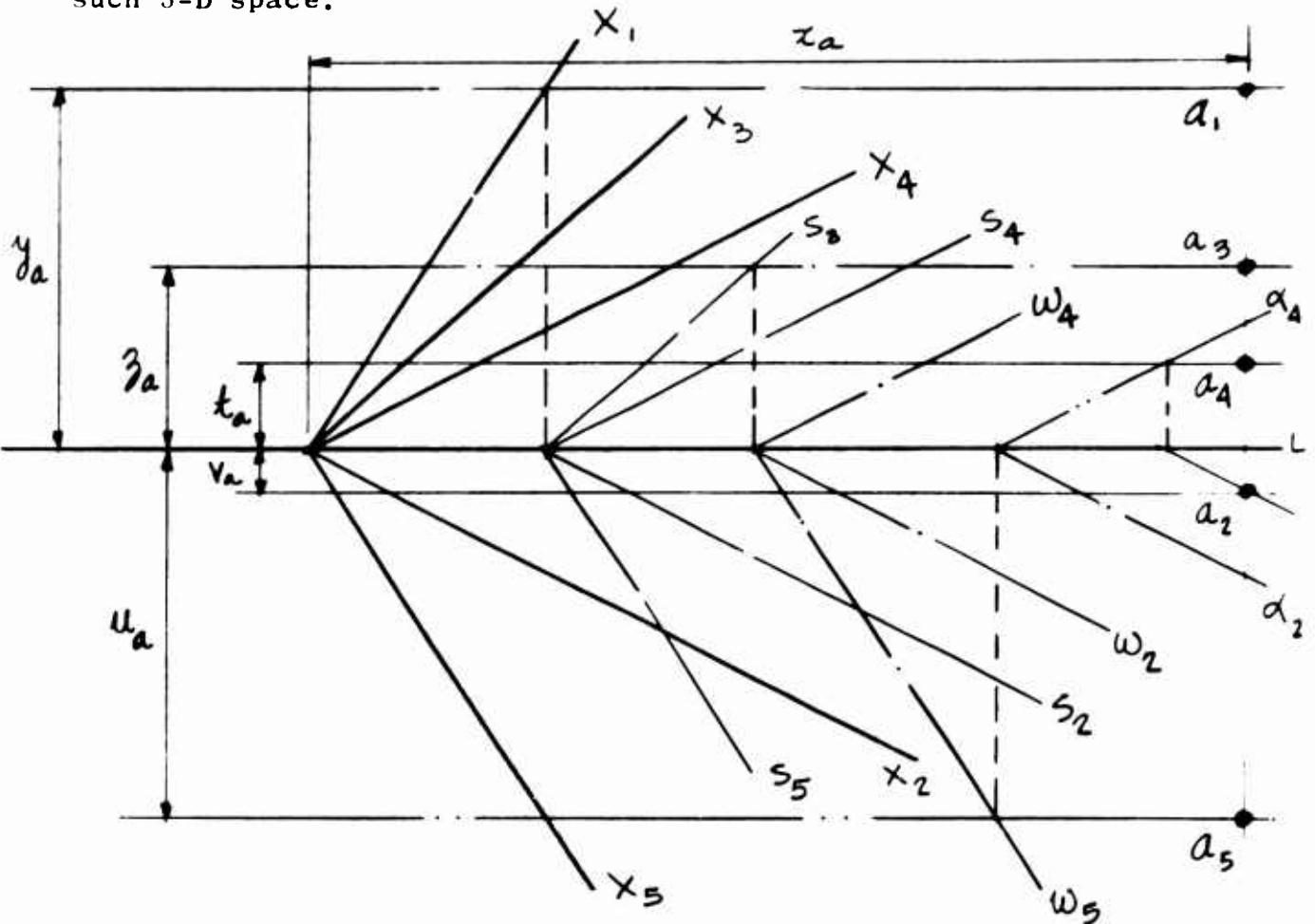


Figure 31

Let us assume now the second possibility, when there is no one coordinate characterizing a group of points. As we said, if the number of coordinates is five, the only alternative is to use a 5-D space which is to be represented, in orthographic projections, according to a six-dimensional descriptive geometry. Figure 32 shows the representation of one such 5-D space.



5 -D space (X)

Figure 32



COMMENTS AND CONCLUSION

It is clear that the possibility of representing graphically the relationships among points characterized by an arbitrary number of variables or parameters, is unlimited.

A very clear pattern can be seen of how the descriptive geometry of more than three dimensions can be developed. In fact, it all seems to be just a matter of adding more lines to the representation of a given "space" to obtain another "space" with two, three, or more dimensions. Indeed this is the procedure, and we can think of no other example to demonstrate the rigid and logical, and yet simple, relationship among "spaces" of two, three, four, and many dimensions. And all under the light of the descriptive geometry.

We would not attempt, naturally, to explore the graphical properties to be found in a five or in a six-dimensional space, through studies in the corresponding descriptive geometry. However, we want to reinforce the fact that the relationships concerning to the conditions of belonging among the geometric elements, remain unaltered in each multi-dimensional space. Thus, by properly relating these geometric elements, we will find that the properties established for a 3-D space, through studies in four-dimensional descriptive geometry, are found unaltered in any 3-D space located within a 4-D space. And so on.

By looking at the constructions showing such multi-dimensional spaces, one can realize that the simplicity of the method used in maintaining these relationships can be easily expressed through analytical expressions. In fact, by working from the orthographic projections, where all graphical constructions express the true relations in multi-dimensional space, the analytical geometry to be used should be simply two-dimensional. This affirmation may sound a little bold. May be so. But let us remember that the relations in multi-dimensional space are already established and indicated through a graphical language, in a two-dimensional way: in the plane. All we have to do is to properly express these relations, in the plane. And for that we need no multi-dimensional analytic geometry.

If anyone doubts it, give it a try.

CASE FILE

PRINCETON UNIVERSITY  
SCHOOL OF ENGINEERING AND APPLIED SCIENCE  
PRINCETON, NEW JERSEY 08540

September 22, 1965.

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Research Engineer to our department:

Four-Dimensional Descriptive Geometry Rotation -  
Descriptive Solution, March 1965.

Four-Dimensional Descriptive Geometry Problems on  
3-D Spaces, March 1965.

Graphical Plotting of Data. An Application of Three  
and Four Dimensional Theoretical Descriptive Geometries,  
March 1965.

Four-Dimensional Descriptive Geometry Metric Problems:  
Angles - Descriptive Solution, May 1965.

Four-Dimensional Descriptive Geometry Proposition of a  
Problem, June 1965.

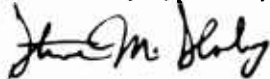
Four-Dimensional Descriptive Geometry Metric Problems:  
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Four-Dimensional Descriptive Geometry Proposed Problems,  
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